Gravitationally Induced Acceleration of Spheres in a Creeping Flow—A Heat Transfer Analogy

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In a recent paper (*1*), consideration is given to transient creeping flow around spheres. The expressions for the sphere velocity studied therein are derived in this note via the utility of an equation of motion for a sphere (as opposed to that of a fluid). In addition, an analogy is discussed between velocity of a sphere for a gravitationally induced motion and temperature of a sphere situated in conductive or convective environments.

The equation of motion for a solid sphere is well known, for example, (2)

$$(m + m_f/2)\frac{du}{dt} = (m - m_f)g - \frac{9m_f}{2a^2}\nu_f u - \frac{9m_f}{2a}\sqrt{\frac{\nu_f}{\pi}}\int_0^t \frac{du}{d\xi}\frac{d\xi}{\sqrt{t-\xi}} \quad (1)$$

In terms of dimensionless variables V and τ, Equation (1) is recast as

$$\frac{dV}{d\tau} = 1 - \beta V - \frac{\beta}{\sqrt{\pi}}\int_0^\tau \frac{dV}{d\xi}\frac{d\xi}{\sqrt{\tau-\xi}} \quad (2)$$

Clearly, for flows with negligible acceleration drag, Equation (2) provides the known solution $V = [1 - \exp(-\beta\tau)]/\beta$, for a solid sphere initially at rest. The general solution of (2), taking the acceleration drag into account, is obtained by means of Laplace transform

$$\overline{V} = \frac{1}{p(p + \beta\sqrt{p} + \beta)} \quad (3)$$

the solution being in agreement with that given in (*1*). After a slight arrangement, the equation satisfying $\overline{V}$ can be written as

$$p^2\overline{V} + (2\beta - \beta^2)p\overline{V} + \beta^2\overline{V} = 1 - \beta/\sqrt{p} + \beta/p \quad (4)$$

and in turn, that governing V as

$$\frac{d^2V}{d\tau^2} + (2\beta - \beta^2)\frac{dV}{d\tau} + \beta^2 V = \beta - \frac{\beta}{\sqrt{\pi\tau}} \quad (5)$$

subject to $V(0) = 0$ and $V'(0) = 1$.

The differential equation governing the velocity of a gas bubble can be developed in a manner similar to the foregoing. In this case, however, not only the initial velocity (taken to be zero) and the initial acceleration, but also the initial rate of change of acceleration must be known. The velocity $\overline{V}$ (in the Laplace transform plane) of a gas bubble is governed by (*1*)

$$\overline{V} = \frac{\sqrt{p} + 3}{p[p^{3/2} + 3p + 2\beta(\sqrt{p} + 1)]} \quad (6)$$

and, in turn, it can be shown that V satisfies the following differential equation:

$$\frac{d^3V}{d\tau^3} + (4\beta - 9)\frac{d^2V}{d\tau^2} + (4\beta^2 - 12\beta)\frac{dV}{d\tau} - 4\beta^2 V = \frac{4\beta}{\sqrt{\pi\tau}} - 6\beta \quad (7)$$

with $V(0) = 0$, $V'(0) = 1$ and $V''(0) = -2\beta$.

The availability of Equations (5) and (7) make it possible to readily compute (with the help of a digital computer) the velocity histories of a solid sphere or of a gas bubble.

Consider now a sphere at a certain uniform temperature situated in a surrounding medium of a very large extent. The initial temperature of the surroundings is the same as that of the sphere. It is assumed that the sphere is heated internally (with a spatially uniform rate) and that thermal conductivity of the sphere is high relative to that of the surrounding medium. The sphere temperature thus varies only temporally. It can be shown in a straightforward way that the dimensionless temperature $\theta(\tau = \alpha_f t/a^2)$ in the Laplace transform plane is governed by

$$\left(\frac{\alpha_f mcT_0}{Q_a^2}\right)\overline{\theta} = \frac{1}{p(p + \lambda_3\sqrt{p} + \lambda_3)} \quad (8)$$

A comparison of Equations (3) and (8) reveals that the velocity of a solid sphere and the temperature of a perfectly conducting sphere are governed by a similar equation. In fact, when the parameter β (of the fluid flow problem) and the parameter λ_3 (of the heat transfer problem) are equal, then the dimensionless velocity V is identically equal to $(\alpha_f mcT_0\theta)/(Qa^2)$.

A similar analogy can be established between a velocity of a gas bubble and a temperature of a solid sphere. Toward this end, consider transfer of heat from a perfect spherical conductor by convection (with a constant heat transfer coefficient h). In this case, as in the foregoing one, the sphere is uniformly heated and the initial temperature is the same as that of the surrounding fluid. It is known (*3*) that the dimensionless sphere temperature in the Laplace transform plane can be written as

$$\left(\frac{\alpha_f mcT_0}{Qa^2}\right)\overline{\theta} = \frac{\sqrt{p} + \lambda_1 + 1}{p[p^{3/2} + (1 + \lambda_1)p + \lambda_2(\sqrt{p} + 1)]} \quad (9)$$

which agrees in form with the expression (6). In fact, when $1 + \lambda_1 = 3$ and $\lambda_2 = 2\beta$, then $(\alpha_f mcT_0\theta)/(Qa^2) = V$. The author proposes to name the aforementioned analogies as ES principles.

NOTATION

a = radius of sphere
c = specific heat
m = mass of sphere
m_f = mass of displaced fluid
p = Laplace transform parameter
Q = rate of heat generation in the sphere
t = time
T = temperature
T_0 = initial temperature
u = sphere velocity
V = dimensionless velocity of sphere, $V = u/[\beta$ (Stokes velocity)]

Greek Letters

α = thermal diffusivity
β = density ratio parameter, $9\rho_f/[2(\rho + \rho_f/2)]$
γ = density and specific heat ratio parameter, $\gamma = 3(\rho_f c_f)/(\rho c)$
θ = dimensionless temperature, $\theta = (T - T_0)/T_0$
$\lambda_1, \lambda_2, \lambda_3$ = heat transfer parameters, $\lambda_1 = ha/k_f$, $\lambda_2 = \frac{4\pi a^3 \rho_f c_f}{mc} \frac{ha}{k_f}$, $\lambda_3 = \frac{4\pi a^3 \rho_f c_f}{mc}$
ν = kinematic viscosity
ρ = density
τ = dimensionless time, $\tau = \nu t/a^2$ (fluid flow problem) and $\tau = \alpha_f t/a^2$ (heat transfer problem)

Subscripts and Superscripts

f = outer flow quantities
— = Laplace transformation

LITERATURE CITED

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Axial Dispersion of a Non-Newtonian Liquid in a Packed Bed

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Fluid behavior in a packed bed may be closely approximated by the socalled dispersion model. The degree of fluid mixing based on this model may be expressed in terms of Peclet number defined as $\bar{u}d_p/E$. Recently a correlation between N_{Pe} and N_{Re} for Newtonian fluid through packed bed was obtained by Chung and Wen (*2, 3*) as

$$\epsilon N_{Pe} = 0.2 + 0.011\ (N_{Re})^{0.48}$$

The dispersion coefficients can be conveniently obtained by a pulse-response technique. Hougen and Walsh (*6*) presented an extensive discussion of theoretical and practical aspects for the analysis of pulse data. To obtain the model parameters from the data of pulse testing, Johnson (*7*) used: (1) frequency response analysis, (2) the moment method analysis, and (3) the s-plane analysis. Clements and Schnelle (*4*) discussed the normalized frequency contents of various shapes of pulses. They showed that sharp pulses are needed to obtain high frequency content in the signal. Justice (*8*) used pulse techniques to study longitudinal dispersion in a packed bed with and without mass transfer.

EXPERIMENT

A schematic diagram of the apparatus used is shown in Figure 1. Water and 0.1 and 0.35% aqueous solutions of Polyox 301 were fed from a reservoir to the bottom of a packed bed filled with glass beads. The glass beads had diameters of 3/16 and 9/16 in. The tracer was the sodium salt of fluorescein ($Na_2C_{20}H_{10}O_5$, di-sodium salt of 9-0-carboxyphenyl-6-hydroxy-3-isoxanthene). The tracer transmitted light of wavelength in the range of 5,000 to 8,000 Å. A linear relationship exists between the concentration and the intensity of light transmitted by the tracer. Two sets of detecting apparatus (including a phototube, a recorder and a high voltage power supply) were mounted at the bottom and the top of the packed section to detect the input and output concentration of the tracer. Tracer was injected into the calming section very

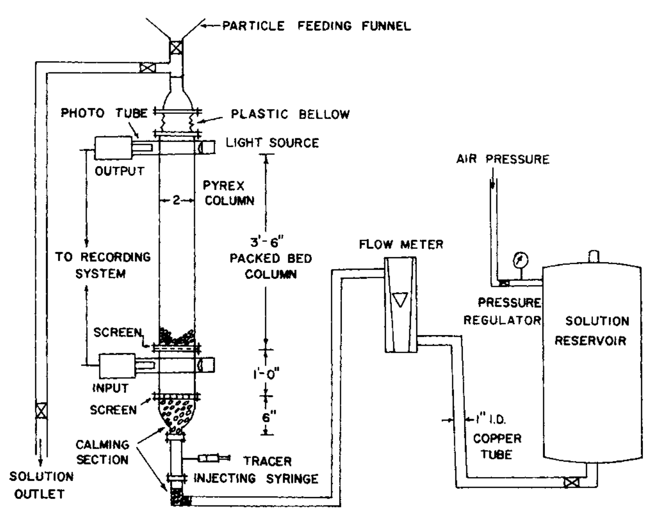

Fig. 1. Schematic diagram of the equipment for pulse response technique on packed bed system.